4/EH-29 (iv) (Syllabus-2015)

2017

(April)

MATHEMATICS

(Elective/Honours)

(Statics and Dynamics)

(GHS-41)

Marks : 75

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **five** questions, taking one from each Unit.

Answer Statics and Dynamics in two separate books

UNIT—I

1. (a) State Lami's theorem. Forces P, Q, R acting along $\overrightarrow{IA}$, $\overrightarrow{IB}$, $\overrightarrow{IC}$, where I is the incentre of the triangle ABC, are in equilibrium. Show that

$$P : Q : R = \cos\frac{1}{2}A : \cos\frac{1}{2}B : \cos\frac{1}{2}C$$ 1+4=5

(b) Two forces P and Q acting at a point have got a resultant R; if Q be doubled, R is doubled. Again, if Q be reversed in direction, then also R is doubled. Show that

$$P:Q:R=\sqrt{2}:\sqrt{3}:\sqrt{2}$$ 5

(c) Three forces P, Q, R act in the same sense along the sides $\overrightarrow{BC}$, $\overrightarrow{CA}$, $\overrightarrow{AB}$ of a triangle ABC. Show that, if their resultant passes through the incentre, then $P+Q+R=0$. 5

2. (a) Show that the resultant of three equal like parallel forces acting at the angular points of a triangle passes through the centroid of the triangle. 5

(b) Show that the algebraic sum of the moments of the forces forming a couple about any point in their plane is a non-zero constant and equal to the moment of the couple. 5

(c) Define the moment of a force about a point. Show that the magnitude of the moment of a force about a point is represented by twice the area of the triangle formed by joining the point to the extremities of the line representing the force. 1+4=5

UNIT—II

3. (a) Forces of magnitudes 2, 4, 6, 8, $8\sqrt{2}$ act along the sides $\overrightarrow{AB}$, $\overrightarrow{BC}$, $\overrightarrow{CD}$, $\overrightarrow{DA}$ and the diagonal $\overrightarrow{BD}$ of a square of side 2 units in the senses indicated by the order of the letters. Taking $\overrightarrow{AB}$, $\overrightarrow{AD}$ as the axes of x and y respectively, find the magnitude of the resultant force. 5

(b) A heavy rod is suspended from a point O by two strings OA and OB. Show that the plane OAB is vertical. 5

(c) How high can a particle rest inside a hollow sphere of radius a, if the coefficient of friction be $\frac{1}{\sqrt{3}}$? 5

4. (a) Find the centre of gravity of a uniform triangular lamina. 5

(b) $ABCD$ is a lamina in the form of a trapezium in which AB and CD are parallel and of lengths a and b respectively. Prove that the distance of the centre of gravity of $ABCD$ from the side AB is

$$\frac{h}{3}\cdot\frac{a+2b}{a+b}$$

h being the height of the trapezium. 6

(c) Define the following : 2×2=4

(i) Coefficient of friction

(ii) Angle of friction

UNIT—III

5. (a) At what distance from the centre will the velocity of a particle, executing simple harmonic motion, of amplitude a, be half of the maximum? 3

(b) A particle moves in a straight line. Its acceleration, directed towards a fixed point O in the line, is equal to $\mu\left(\frac{a^5}{x^2}\right)^{1/3}$, when it is at a distance x from O. If it starts from rest at a distance a from O, then show that it will arrive at O after a time $\frac{8}{15}\sqrt{\frac{6}{\mu}}$. 6

(c) Two smooth elastic spheres of masses m_1 and m_2 impinge directly when moving along the same line with velocities u_1 and u_2 respectively. Calculate the loss in kinetic energy due to the impact. 6

6. (a) It is known that earth attracts a body outside its surface with a force varying inversely as the square of the distance from the centre. A particle is attracted towards the centre of the earth, starting from rest of infinity. Find the velocity of the particle on reaching the earth's surface. 5

(b) A sphere impinges obliquely on another sphere at rest. If the two spheres are smooth, perfectly elastic and equal in mass, prove that they move at right angles to each other after impact. 5

(c) A particle of mass m is repelled with a force $m\mu x$, where x is its distance from the source of repulsion O. If the particle starts from rest at a distance a from O, where $x \geq a$; find its distance from O at time t. 5

UNIT—IV

7. (a) A particle is projected vertically upwards with velocity u in a medium whose resistance varies as the velocity. Find the greatest height attained by the particle. 5

(b) If at any point of the parabolic path of a projectile, the velocity be u and the inclination to the horizontal be θ, show that the particle is moving at right angles to its former direction after a time $\frac{u}{g}\operatorname{cosec}\theta$. 5

(c) A body is projected at an angle α to the horizon, so as just to clear two walls of equal height a at a distance $2a$ from each other. Show that the range is equal to $2a\cot\frac{\alpha}{2}$. 5

8. (a) The fluid resistance offered to the motion of a ship is given by the formula $R = av + bv^2$, where a and b are constants. The propulsion of a ship, weighing w, is stopped at the instant when $v = v_0$. Find the distance the ship will then move before coming to rest. 5

(b) A particle, of mass m, is projected vertically under gravity, the resistance of the air being mk times the velocity. Show that the time of ascent to the greatest height is $\frac{V}{g}\log(1+\lambda)$, where V is the terminal velocity of the particle and λV is its initial velocity. 5

(c) A particle is projected with velocity u at an angle α to the horizontal. Find the horizontal range of the particle. 5

UNIT—V

9. (a) Deduce the following work-energy equation, 'change in kinetic energy = work done by the forces', for a particle moving in a smooth plane curve under the action of conservative forces. 7

(b) A particle is projected from the lowest point of a smooth vertical circle of radius a with velocity u and moves along the inside of it. Apply the principle of energy to find the velocity of the particle at any angular distance θ from the lowest point. 4

(c) A heavy particle slides down a smooth cycloid (intrinsic equation $s = 4a\sin\psi$), starting from rest at a cusp, the axis being vertical and vertex downwards. Prove that the magnitude of the acceleration is equal to g at every point of the path. 4

10. *(a)* A point describes the cycloid $s = 4a \sin \psi$ with uniform speed u. Find its acceleration at any point in terms of u, a and s. 3

(b) A heavy particle of weight W, attached to a fixed point by a light inextensible string, describes a circle in a vertical plane. The tension in the string has values mW and nW respectively when the particle is at the highest and the lowest points of its path. Show that $n = m + 6$. 6

(c) A shot of mass m is fired from a gun of mass M with a velocity u relative to the gun. Show that the actual velocities of the shot and the gun are $\frac{Mu}{M+m}$ and $\frac{mu}{M+m}$ respectively and that their kinetic energies are inversely proportional to their masses. 6

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